Quasi-TEM Study of Microshield Lines with Practical Cavity Sidewall Profiles

Kwok-Keung M. Cheng, *Member, IEEE,* and Ian D. Robertson, *Member, IEEE*

***Abstract*— This paper presents the quasi-TEM characteristics of microshield lines with practical cavity sidewall profiles. A conformal mapping method is used for the derivation of the electrical parameters of the structures. In this study, numerical results for the characteristic impedances of air-suspended microshield lines with both positive and negative sidewall slopes are presented. Simple and explicit CAD-oriented expressions are proposed for the design and analysis of rectangular-shaped microshield line. Comparisons are made between the results obtained by these formulas and by a standard numerical technique. Furthermore, the sensitivities of the electrical parameters of a rectangular-shaped microshield line to an imperfect sidewall etching process, leading to nonvertical sidewall profiles, are also examined.**

I. Introduction

RECENTLY, the microshield line, a new type of transmission line [1], has been the subject of growing interest as it has presented a solution to technical and technological problems encountered in the design of microstrip and coplanar lines. The microshield line, when compared with the conventional ones, has the ability to operate without the need for via-holes or the use of air-bridges for ground equalization. There are further advantages, like reduced radiation loss, reduced electromagnetic interference, and the availability of a wide range of impedance. In this configuration, the ground plane has been deformed from its original planar form to totally or partially surround the inner strip conductor. This structure can be fabricated monolithically using dielectric etching and metal deposition techniques [2]. The inner conductor can be printed on a thin substrate layer or suspended in air by using membrane technology. Microshield lines using the membrane technology exhibit TEM dispersionless characteristics over a very broad frequency range. Various types of microshield structures have been reported [1], [4], [5], including rectangular, V, elliptic, and circular-shaped transmission lines. Theoretical studies of a rectangular-shaped microshield line have been performed both by a method of moments [2] and a conformal mapping technique [3].

In this paper, the following developments are presented:

1) An analytical solution for the characteristic impedance of a microshield line with imperfectly etched cavity sidewalls (Fig. 1), using a conformal mapping technique.

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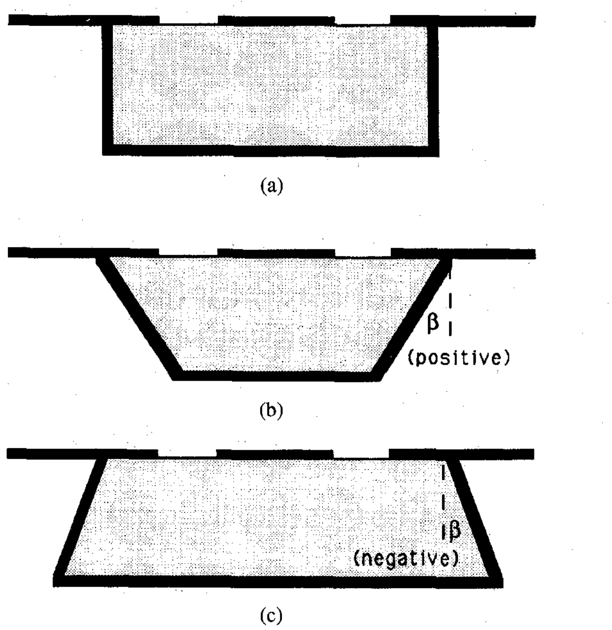

Fig. 1. Microshield line. (a) Rectangular-shaped. (b) and (c) Practical profiles, positive or negative slope depending upon the orientation and etching conditions.

2) A set of simple and explicit formulas for the computation of the characteristic parameters of a rectangular-shaped microshield line.
3) Line parameter sensitivity analysis of a rectangular-shaped microshield line fabricated by an imperfect sidewall etching process.

The characterization of the above type of microshield line is extremely important because it could offer additional flexibility in the design of integrated circuits. Furthermore, it allows one to evaluate the actual characteristics of a microshield line normally designed to be rectangular-shaped, but the fabrication of which is imperfect. As pointed out in [2], the wet etching performed during the fabrication of the membrane is anisotropic, yielding cavities with steep sidewalls, with either a positive or negative slope, as depicted in Fig. 1, depending on the line's orientation and the etching conditions. Previously, the assumption of vertical cavity sidewalls has been employed in the analysis of the microshield line [2], [3]. The significance of the sensitivity analysis presented here is therefore to examine the conditions for which the above approximation is acceptable and to quantify the effect of the practical nonvertical sidewalls on the electrical parameters of the structures.

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II. Analysis

The configuration to be studied is shown in Fig. 2(a), where the lower ground plane is bent within the dielectric to form a trapezoidal cavity. All metallic conductors are assumed to be infinitely thin and perfectly conducting, and the upper ground planes to be sufficiently wide as to be considered infinite in the model. It is assumed that the air-dielectric boundary between the center conductor and the upper ground plane behaves like a perfect magnetic wall. This ensures that no electric field lines emanating into the air from the center conductor cross the air-dielectric boundary. Although this assumption is hardly verified for large slots, it has been proved to yield excellent results for practical line dimensions [7]. Transverse symmetry is assumed so that no antisymmetric mode is excited. The center conductor, of width $2a$, is placed between the two upper ground planes, of spacing $2b$, which are located on a substrate of thickness h, with relative permittivity ε_r. The overall capacitance per unit length of the line can therefore be considered as the sum of the capacitance of the upper region (air) and the lower region (dielectric). The capacitance of the lower region can be evaluated through a suitable sequence of conformal mappings. First, the interior of the lower region is mapped onto the t domain (Fig. 2) by the Schwartz-Christoffel transformation

$$z = A \int_0^t (t^2 - 1)^{-p} (t^2 - t_c^2)^{p-1} \quad dt \tag{1}$$

and then back onto the w domain using a second mapping function

$$w = \int_0^t \frac{dt}{\sqrt{(t^2 - t_a^2)(t^2 - t_b^2)}} \tag{2}$$

where

$$0 < t_a < t_b \le t_c < 1$$
$$-\frac{\pi}{2} < \beta < \arctan\left(\frac{W}{2h}\right)$$

and $p = \frac{1}{2} - \frac{\beta}{\pi}$. Note that the angle β (in radian) can either be positive or negative to account for the two possible cavity sidewall profiles [Fig. 1(b) and (c)]. The intermediate parameters t_a, t_b, and t_c, are evaluated by the expressions shown in (3), as a function of β and geometrical ratios, $a/h, b/h$, and W/h, of the structure.

$$\frac{2h}{W}\Delta = \cos\beta \int_0^{\operatorname{arccosh}(1/t_c)} \frac{d\theta}{\sinh^{1-2p}\theta(1 - t_c^2\cosh^2\theta)^p} \tag{3a}$$

$$\frac{2a}{W}\Delta = \int_0^{\arcsin(t_a/t_c)} \frac{d\theta}{\cos^{1-2p}\theta(1 - t_c^2\sin^2\theta)^p} \tag{3b}$$

$$\frac{2b}{W}\Delta = \int_0^{\arcsin(t_b/t_c)} \frac{d\theta}{\cos^{1-2p}\theta(1 - t_c^2\sin^2\theta)^p}$$

$$\Delta = \int_0^{\pi/2} \frac{d\theta}{\cos^{1-2p}\theta(1 - t_c^2\sin^2\theta)^p}. \tag{3c}$$

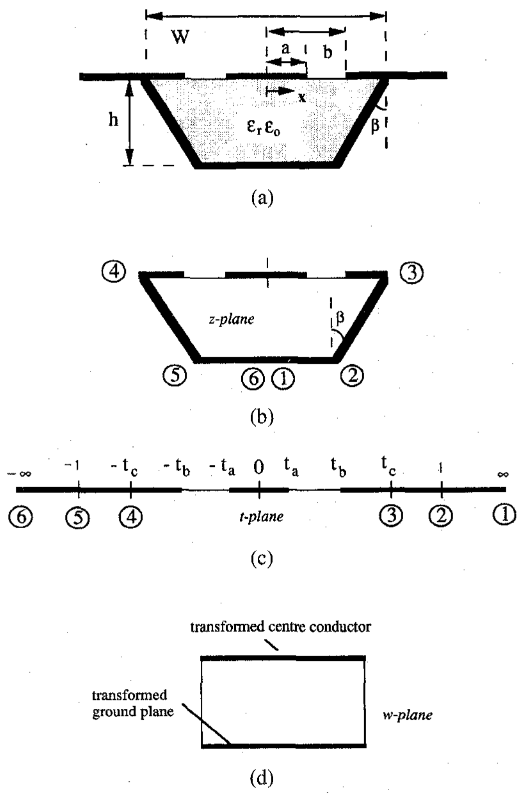

Fig. 2. Conformal mapping of microshield line with nonvertical cavity sidewalls.

If the total capacitances of the upper and lower regions are referred to as C_1 and C_2, respectively, then the overall capacitance per unit length of the line is given by [7]

$$C_T(\varepsilon_r) = C_1 + C_2 = 2\varepsilon_0 \frac{K(k_1)}{K'(k_1)} + 2\varepsilon_r\varepsilon_0 \frac{K(k_2)}{K'(k_2)}. \tag{4}$$

$K(k)$ is the complete elliptic integral of the first kind, $k_1 = a/b$, and $k_2 = t_a/t_b$. Hence, the effective permittivity and the characteristic impedance of the line are, respectively

$$\varepsilon_{\text{eff}} = \frac{C_T(\varepsilon_r)}{C_T(1)} \tag{5}$$

$$Z_0 = v_0^{-1}[C_T(\varepsilon_r)C_T(1)]^{-\frac{1}{2}} \tag{6}$$

where v_0 is the speed of light in free space. The line parameters can therefore be calculated from (4), (5), and (6), using the simple formulas of Hilberg [8] for the ratio $K(k)/K'(k)$. Note that both the rectangular and V-shaped microshield line are actually special cases of the general trapezoidal configuration where $\beta = 0$ and $\beta = \arctan(W/2h)$, respectively.

III. Rectangular-Shaped Microshield Line ($\beta = 0$)

As previously mentioned, the wet etching performed during membrane formation is anisotropic, which yields cavities with nonvertical sidewalls. However, the slopes of these sidewalls can be minimized (to less than a few degrees) with a more sophisticated etching processes. Hence, for the sake of simplicity in the analysis and without much loss of accuracy (will be addressed in Sections IV and V), the cavities of these microshield lines may thus be considered as rectangular-shaped ($\beta = 0$). The expressions in (3a), (3b), and (3c) can therefore be reduced to

$$t_c = \left(\frac{e^{\frac{W}{2h}\pi} - 2}{e^{\frac{W}{2h}\pi} + 2}\right)^2 \quad \text{for } 1 < \frac{W}{2h} < \infty$$

$$t_c = \sqrt{1 - \left(\frac{e^{\frac{2h}{W}\pi} - 2}{e^{\frac{2h}{W}\pi} + 2}\right)^4} \quad \text{for } 0 < \frac{W}{2h} < 1 \qquad (7a)$$

$$\frac{2a}{W} = \frac{F(\arcsin(t_a/t_c), t_c)}{K(t_c)} \qquad (7b)$$

$$\frac{2b}{W} = \frac{F(\arcsin(t_b/t_c), t_c)}{K(t_c)} \qquad (7c)$$

where $F(\phi, k)$ is the incomplete elliptic integral of the first kind, written in Jacobi's notation, which can be evaluated using the routine in [10]. The above formulas have been presented in [3]. One disadvantage of these expressions is that they are implicit, which makes them difficult to use. Hence, it is helpful to have an analytical formula which is both simple and explicit. Such a formula will now be presented.

The incomplete elliptic integral function in the above expressions can be evaluated with high accuracy by the following procedure in which closed-form expressions are readily available

for $0 \le \phi \le \alpha$

$$F(\phi, k) = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}} \approx \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2\theta^2}} = \frac{1}{k}\arcsin(k\phi) \qquad (8a)$$

where

$$\alpha = \frac{\pi}{4} - \frac{\beta}{4}$$
$$\beta = \sqrt{\pi^2 - 8}$$

for $\alpha \le \phi \le \frac{\pi}{2} - \alpha$

$$F(\phi, k) = \int_0^{\alpha} \frac{d\theta}{\sqrt{1 - k^2\sin^2\theta}} + \int_{\alpha}^{\phi} \frac{d\theta}{\sqrt{1 - k^2\sin^2\theta}} \approx \frac{1}{k}\arcsin(k\alpha) + \int_{\alpha}^{\phi} \frac{d\theta}{\sqrt{1 - k^2[0.5 + 2\alpha(\theta - \pi/4)]}} \qquad (8b)$$

$$= \frac{1}{k}\arcsin(k\alpha) + \frac{1}{\alpha k^2}\left\{\sqrt{1 - \frac{k^2}{2}(1 - \alpha\beta)} - \sqrt{1 - \frac{k^2}{2}[1 + 4\alpha(\phi - \pi/4)]}\right\}$$

for $\frac{\pi}{2} - \alpha \le \phi \le \frac{\pi}{2}$

$$F(\phi, k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - k^2\sin^2\theta}} - \int_{\phi}^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - k^2\sin^2\theta}} \approx \frac{S(k)}{k} - \int_{\phi}^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1 - k^2[1 - (\pi/2 - \phi)^2]}} = \frac{S(k)}{k} - \frac{1}{k}\text{arcsinh}\left\{\frac{k}{\sqrt{1 - k^2}}\left(\frac{\pi}{2} - \phi\right)\right\} \qquad (8c)$$

where

$$S(k) = \arcsin(k\alpha) + \text{arcsinh}\left(\frac{k\alpha}{\sqrt{1 - k^2}}\right) + \frac{1}{\alpha k}\left\{\sqrt{1 - \frac{k^2}{2}(1 - \alpha\beta)} - \sqrt{1 - \frac{k^2}{2}(1 + \alpha\beta)}\right\}.$$

Substituting (8) into (7), and after some manipulations, the following formulas can be found

for $0 \le \frac{2x}{W} \le \delta_1 = \frac{1}{S(t_c)}\arcsin(t_c\alpha)$

$$\frac{t}{t_c} = \sin\left\{\frac{1}{t_c}\sin\left[\frac{2x}{W}S(t_c)\right]\right\} \qquad (9a)$$

for $\delta_1 \le \frac{2x}{W} \le \delta_2 = 1 - \frac{1}{S(t_c)}\text{arcsinh}\left(\frac{t_c\alpha}{\sqrt{1 - t_c^2}}\right)$

$$\frac{t}{t_c} = \sin\left\{\frac{\pi}{4} + \frac{1}{2\alpha}\left(\frac{1 - \gamma^2}{t_c^2} - \frac{1}{2}\right)\right\} \qquad (9b)$$

$$\gamma = \sqrt{1 - \frac{t_c^2}{2}(1 - \alpha\beta) - t_c\alpha S(t_c)\left\{\frac{2x}{W} - \delta_1\right\}}$$

for $\delta_2 \le \frac{2x}{W} \le 1$

$$\frac{t}{t_c} = \cos\left\{\frac{\sqrt{1 - t_c^2}}{t_c}\sinh\left[\left(1 - \frac{2x}{W}\right)S(t_c)\right]\right\}. \qquad (9c)$$

It can be observed from (9) that the values of t_a and t_b are now given explicitly in terms of elementary functions of the geometrical parameters $a/h, b/h$ and W/h.

IV. Sensitivities Analysis

In this section, a sensitivity approach for investigating the effect of an imperfect sidewall etching process on the characteristics of a rectangular-shaped microshield line is given. For the sake of brevity, only the sensitivity of the lower region capacitance (C_2) with respect to sidewall angle β is shown here

$$\frac{1}{C_2}\frac{dC_2}{d\beta}\bigg|_{\beta=0} = \frac{k_2^2}{\pi\gamma(k_2)}\{\psi(a, t_a) - \psi(b, t_b)\} \qquad (10a)$$

$$\gamma(k) = (1 - k)k^{3/2}\ln\left(2\frac{1 + \sqrt{k}}{1 - \sqrt{k}}\right) \quad \text{for } 0.707 < k < 1$$

$$\gamma(k) = \gamma(k') \qquad \text{for } 0 < k < 0.707 \qquad (10b)$$

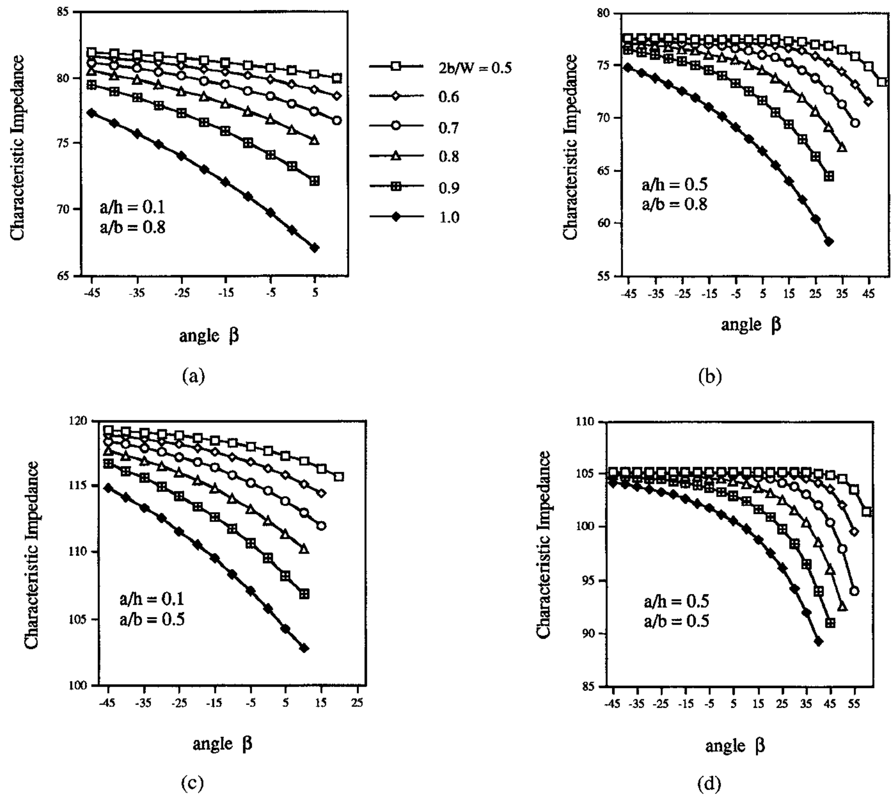

Fig. 3. Plots of the characteristic impedance (in ohm) of a practical air-suspended microshield line versus β (in degree), as a function of geometrical ratios $a/h, a/b$, and $2b/W$.

$$\psi(x,t) = (1-t^2)^{\frac{1}{2}}\left(\frac{t_c^2}{t^2}-1\right)^{\frac{1}{2}} \times \left\{\frac{\gamma(t_c)}{K(t_c)}\left[\frac{2x}{W}\xi(t_c)-\xi(t)\right]\left[\frac{W}{2h}\eta-\zeta(t_c)\right] + \frac{2x}{W}\zeta(t_c)-\zeta(t)\right\}$$

$$\xi(t) = \int_0^{\arcsin(t/t_c)} \frac{\sin^2\theta}{(1-t_c^2\sin^2\theta)^{3/2}}\,d\theta$$

$$\zeta(t) = \int_0^{\arcsin(t/t_c)} \frac{\ln\left(\frac{1-t_c^2\sin^2\theta}{\cos^2\theta}\right)}{(1-t_c^2\sin^2\theta)^{1/2}}\,d\theta$$

$$\eta = \int_0^{\operatorname{arccosh}(1/t_c)} \frac{\ln\left(\frac{1-t_c^2\cosh^2\theta}{\sinh^2\theta}\right)}{(1-t_c^2\cosh^2\theta)^{1/2}}\,d\theta. \quad (10c)$$

The parameters t_a, t_b, and t_c are obtained by solving (7) or (9). The numerical accuracy of these sensitivity calculations has been verified by comparing the results obtained by the presented method with those obtained by the method of perturbation.

V. Discussions

Examples of design curves of an air-suspended microshield line with nonvertical cavity sidewalls are given in Fig. 3. They are obtained by solving (3)–(6), using an iterative numerical technique [10]. The effect of the membrane is neglected in the analysis since its thickness is small compared to the cavity height. However, it should be noted that due to the finite thicknesses of the membrane, the effective dielectric constant is slightly larger than 1 since a fraction of the fields will be contained within the thin dielectric layers [11]. Fig. 3 show how the characteristic impedance of the microshield line varies with geometrical parameters $2b/W, a/h$ and b/h, for different values of β. For a given shape ratio a/h, b/h, and W/h, it is observed that the characteristic impedance decreases with increasing value of β. This is the result of the increase of the capacitance per unit length of the line. This behavior is explained by the fact that the smaller distance between center conductor and the cavity walls causes the electric flux to be more confined directly underneath the center conductor. However, these impedance reductions are negligible for small values of β and $2b/W$. Note that for a given set of values of $\beta, a/h$, and b/h, maximum decreasing rate occurs at the point when the cavity sidewalls touch the slot edges ($2b/W = 1$).

In Section III, a numerical solution (7) and a set of explicit formulas (9) have been devised for obtaining the characteristic impedance of rectangular-shaped microshield line. In order to examine the accuracy of these formulas, results obtained by the two approaches are tabulated in Table I for comparison. The results show discrepancies in impedance calculations of less than 0.8%, for a wide range of test data.

Finally, plots of the sensitivity coefficient of C_2 for a rectangular-shaped microshield line are depicted in Fig. 4. The coefficient is computed as a function of geometrical ratios a/h and $2b/W$, for two different values of aspect ratio a/b. It is observed that the sensitivity coefficient decreases with decreasing value of b/W and increasing value of a/h. From

TABLE I
COMPARISON OF THE CHARACTERISTIC IMPEDANCE (IN OHM) OF AN AIR-SUSPENDED RECTANGULAR-SHAPED MICROSHIELD LINE, CALCULATED BY A NUMERICAL METHOD AND THE PROPOSED FORMULA IN (9)

a/h	a/b	2b/W = 0.5	2b/W = 0.8	2b/W = 1.0
0.1	0.95	57.34 / 57.34	54.24 / 54.24	45.87 / 45.87
	0.5	118.60 / 117.69	112.34 / 112.34	105.80 / 105.80
0.2	0.95	57.33 / 57.33	54.24 / 54.24	45.87 / 45.87
	0.5	117.43 / 116.66	112.16 / 112.15	105.75 / 105.75
0.5	0.95	56.27 / 56.23	53.94 / 53.94	45.80 / 45.80
	0.5	105.60 / 105.18	104.59 / 104.39	101.22 / 101.13
1.0	0.95	51.73 / 51.68	50.89 / 50.90	44.40 / 44.41
	0.5	84.14 / 84.27	84.19 / 84.25	83.70 / 83.74

Format: Numerical method / Proposed formula

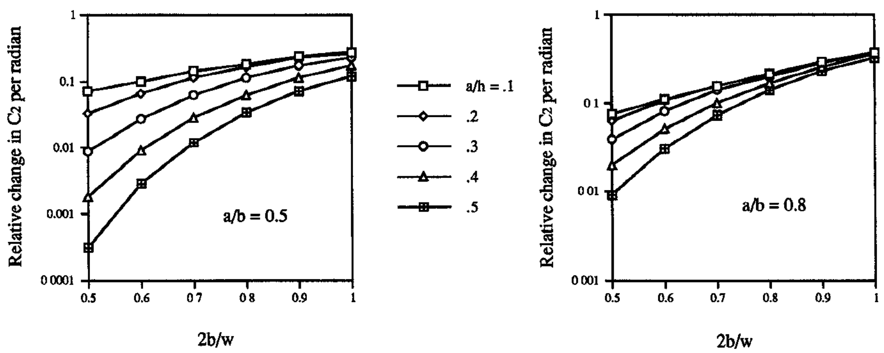

Fig. 4. Plots of cavity capacitance sensitivity as a function of parameters $2b/W$ and a/h.

these graphs, it can be concluded that the discrepancies in impedance calculations between a practical steep-sided and a perfectly rectangular-shaped microshield lines can become very significant under the following conditions: (1) sidewalls are made very close to the slot edges ($2b/W \approx 1$); (2) low impedance value ($a/b \approx 1$); and (3) thick dielectric substrate (large a/h). Note that the frequency range of operation of the microshield line is effectively controlled by the size of the cavity, therefore, the cavity dimensions should also be chosen such that no higher order modes are excited at the desired operating frequency.

VI. CONCLUSION

A conformal mapping method has been described to evaluate the quasi-TEM characteristic parameters of microshield line with practical cavity sidewall profiles. Numerical results have been presented for the characteristic impedance of the line, over a wide range of practical dimensions. The effect of nonvertical cavity sidewalls, due to an imperfect etching process, on the characteristic impedance of the microshield line has also been investigated. It has been observed that this effect is minimized by keeping the cavity sidewalls some distance away from the slot edges and by employing a thin substrate. A simple and explicit formula has been proposed for obtaining the characteristic parameters of a rectangular-shaped microshield line. The numerical accuracy of this expression has been verified by comparing the results with the ones obtained by a numerical method. The analysis procedure shown here can easily be extended to other applications such as asymmetrical lines [6] and coupled lines [9].

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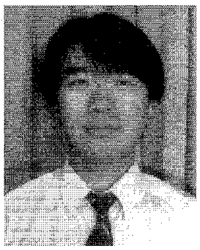

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